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D-003-001308

Seat No. ________

B. Sc. (Sem. III) Examination

March - 2022

Mathematics : Paper - BSMT-301(A)

(Linear Algebra, Calculus & Theory of Equations)

Faculty Code : 003

Subject Code : 001308

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.

(2) Figures to the right side indicate marks of full question.

1 Answer the following questions briefly : **20**

(1) Define binary operation.

(2) Define linear independence.

(3) Define subspace.

(4) Define base of vector space.

(5) If $T : R^3 \to R^3, T(x, y, z) = (x - y, y - z, z - x), \forall (x, y, z) \in R^3$ is linear transformation then find $T(1, 2, 1)$.

(6) Define zero linear transformation.

(7) Define range of a linear transformation.

(8) Define null space.

(9) Define infinite series.

(10) If $\{a_n\}$ and $\{b_n\}$ are divergent then $\{a_n \cdot b_n\}$ is divergent. (True or False)

(11) The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is divergent. (True or False)

(12) Write comparison test for series.

(13) What is the degree of algebraic equation $x^2 - 3x + 6 = 0$?

(14) Find the interval in which the root of equation $x^2 - 2x - 5 = 0$ lies.

(15) Write the iterative formula to find $\sqrt{N}$.

(16) Write the iterative formula to find $\sqrt[3]{N}$.

(17) Define radius of curvature.

(18) Define singular point.

(19) Define point of inflexion.

(20) Define asymptotes.

2 (a) Attempt any **three** out of six : **6**

(1) Check whether set $\{(2,0,0),(0,0,1),(1,0,0)\}$ of vector space R^3 is L.D. or L.I.

(2) Show that set $\{(2,5,-1),(2,4,0)\}$ of R^3 is not a base of R^3.

(3) Show that

$T:R^3 \to R^3, T(x,y,z)=\left(x^2,y^2,z^2\right); \forall (x,y,z)\in R^3$

is not linear transformation.

(4) If U and V are any vector spaces and θ, θ' are zero vectors of U and V respectively, then prove that $T(\theta) = \theta'$.

(5) For linear transformation

$T:R^2 \to R^3, T(x,y)=(x, x+y, y), \forall (x,y)\in R^2$, find N_T.

(6) For linear transformation

$T:R^2 \to R^3, T(x,y)=(x, x+y, y), \forall (x,y)\in R^2$, find R_T.

(b) Attempt any **three** out of six : **9**

(1) Show that intersection of two subspaces W_1 and W_2 of vector space V is also subspace.

(2) $A=\{(1,-2,5),(2,1,-1),(3,-1,b)\}$ is a subset of vector space R^3. If set A is L.D. then find b.

(3) Extend set $A = \{(1,1,1), (2,0,0)\}$ of vector space R^3 to form a basis of R^3.

(4) Find linear transformation $T : R^2 \to R^2$ such that $T(1,2) = (3,4)$ and $T(2,1) = (2,2)$.

(5) If $T : U \to V$ is a linear transformation then show that $R_T = \{T(u) / u \in U\}$ is subspace of V.

(6) Prove that composition of two linear transformation is also a linear transformation.

(c) Attempt any **two** out of five : **10**

(1) Let $T : V \to V$ be any linear transformation such that $T^2 - T + I = 0$ then prove that T is non-singular.

(2) For linear transformation $T : R^3 \to R^3, T(a,b,c) = (a-b+c, b-c, c)$ then find T^{-1}.

(3) In usual notation prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$.

(4) State and prove Rank Nullity theorem.

(5) Show that $A = \{(2,2,3), (2,1,3), (1,0,1)\}$ is base of R^3. Find co-ordinates of $(1,1,1)$ with respect to this base.

3 (a) Attempt any **three** out of six : **6**

(1) Show that the series $1 + 2 + 3 + ... + n + ...$ is divergent.

(2) Show that the series $\sum_1^\infty (-1)^n$ oscillates finitely.

(3) Prove that iterative formula for $\frac{1}{N}$ is $x_{n+1} = x_n(2 - Nx_n)$.

(4) Find the value of $\sqrt{15}$ correct to four decimal places by Newton's iteration method.

(5) Find the radius of curvature $s = c \tan \varphi$.

(6) Prove that $y = ex$ is everywhere concave upwards.

(b) Attempt any **three** out of six : **9**

(1) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n+2}$.

(2) Show that the series $\sum_{1}^{\infty} \sin\frac{1}{n}$ is divergent.

(3) If $P(x) = 2x^4 - 3x^2 + 5x - 2$ then find $P^4(-2)$.

(4) Using Newton's method find $\frac{1}{\sqrt{17}}$ correct to three decimal.

(5) Find the point of inflexion of $x = 3y^2 + y^3$.

(6) Find the asymptotes parallel to co-ordinates for $y^2\left(x^2 - a^2\right) = x$.

(c) Attempt any **two** out of five : **10**

(1) Show that the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + ...$ is convergent but not absolutely convergent.

(2) Find root of $x^3 - x^2 - x - 2000 = 0$ by Horner's method correct to three decimal.

(3) Explain False Position method.

(4) In usual notation prove radius of curvature is $\rho = \frac{\left[1 + y_1^2\right]^{3/2}}{y_2}$.

(5) Show that the radius of curvature at any point on the cardioids is $r = (1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$.